微积分速查表

三角函数

和差化积公式

$$
\begin{aligned}
&[1].\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta \qquad &&[2].\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta\\
&[3].\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta \qquad &&[4].\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta\\
&[5].\tan(\alpha+\beta)=\frac{\tan\alpha\tan\beta}{1-\tan\alpha\tan\beta}
\end{aligned}
$$

积化和差公式

$$
\begin{aligned}
&[1].\sin\alpha\cos\beta=\frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)] \qquad &&[2].\sin\alpha\cos\beta=\frac{1}{2}[\sin(\alpha-\beta)+\sin(\alpha+\beta)]\\
&[3].\cos\alpha\sin\beta=\frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]
\end{aligned}
$$

倍角公式

$$
\begin{aligned}
&[1].\sin 2\alpha=2\sin\alpha\cos\alpha \qquad &&[2].\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=1-2\sin^2\alpha=2\cos^2\alpha-1\\
&[3].\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}
\end{aligned}
$$

极限

两个重要极限

$$
[1].\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad [2].\lim_{x\to\infty}(1+\frac{1}{x})^x=\lim_{t\to 0}(1+t)^{\frac{1}{t}}=e
$$

常用等价无穷小

$$
\begin{aligned}
&[1].\quad \sin x\sim x\quad(x\to 0) &&[2].\quad \tan x\sim x\quad(x\to 0)\\
&[3].\quad \arcsin x\sim x\quad(x\to 0) &&[4].\quad 1-\cos x\sim\frac{1}{2}x^2\quad(x\to 0)\\
&[5].\quad (1+x)^\alpha\sim\alpha x\quad(x\to 0)\quad\alpha\in R &&[6].\quad e^x-1\sim x\quad(x\to 0)\\
&[7].\quad \ln(1+x)\sim x\quad(x\to 0)
\end{aligned}
$$

导数

[1]. $C' = 0 \quad C \in R$

[2]. $(x^{\mu})' = \mu x^{\mu-1}$

[3]. $(a^x)' = a^x \ln a (a > 0且a \neq 1)$

[4]. $(e^x)' = e^x$

[5]. $(\log_a x)' = \dfrac{1}{x \ln a} (a > 0且a \neq 1)$

[6]. $(\ln x)' = \dfrac{1}{x}$

[7]. $(\sin x)' = \cos x$

[8]. $(\cos x)' = -\sin x$

[9]. $(\tan x)' = \sec^2 x$

[10]. $(\sec x)' = \sec x \tan x$

[11]. $(\cot x)' = -\csc^2 x$

[12]. $(\csc x)' = -\csc x \cot x$

[13]. $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$

[14]. $(\arccos x)' = -\dfrac{1}{\sqrt{1-x^2}}$

[15]. $(\arctan x)' = \dfrac{1}{1+x^2}$

[16]. $(arccot \quad x)' = -\dfrac{1}{1+x^2}$

常用泰勒展开

[1]. $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{n=0}^{\infty} x^n \qquad x \in (-1, 1)$

[2]. $\dfrac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots = \sum_{n=0}^{\infty} (-1)^n x^n \qquad x \in (-1, 1)$

[3]. $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} \qquad x \in (-\infty, +\infty)$

[4]. $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} \qquad x \in (-\infty, +\infty)$

[5]. $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n}}{(2n)!} \qquad x \in (-\infty, +\infty)$

[6]. $\tan^{-1} x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \cdots = \sum_{n=0}^{\infty} (-1)^n \dfrac{x^{2n+1}}{2n+1} \qquad x \in [-1, 1]$

[7]. $\ln(x+1) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} + \cdots = \sum_{n=1}^{\infty} (-1)^n \dfrac{x^{n+1}}{n+1} \qquad x \in (-1, 1]$

[8]. $(1+x)^{\alpha} = 1 + \alpha x + \dfrac{\alpha(\alpha-1)}{2!} x^2 + \dfrac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \cdots = \sum_{n=0}^{\infty} C_n^{\alpha} x^n \qquad x \in (-1, 1), \alpha \in R$

基本积分表

[1]. $\int kdx = kx + C$

[2]. $\int x^{\mu} dx = \frac{x^{\mu+1}}{\mu+1} + C(\mu \neq -1)$

[3]. $\int \frac{dx}{x} dx = \ln|x| + C$

[4]. $\int \frac{dx}{1+x^2} dx = \arctan x + C$

[5]. $\int \frac{dx}{\sqrt{1-x^2}} dx = \arcsin x + C$

[6]. $\int \cos dx = \sin x + C$

[7]. $\int \sin x dx = -\cos x + C$

[8]. $\int \sec^2 x dx = \tan x + C$

[9]. $\int \csc^2 x dx = -\cot x + C$

[10]. $\int \sec x \tan x dx = \sec x + C$

[11]. $\int \csc x \cot x dx = -\csc x + C$

[12]. $\int e^x dx = e^x + C$

[13]. $\int a^x dx = \frac{a^x}{\ln a} + C$

[14]. $\int \tan x dx = -\ln|\cos x| + C$

[15]. $\int \cot x dx = \ln|\sin x| + C$

[16]. $\int \sec x dx = \ln|\sec x + \tan x| + C$

[17]. $\int \csc x dx = \ln|\csc x - \cot x| + C$

[18]. $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$

[19]. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln|\frac{x-a}{x+a}| + C$

[20]. $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$

[21]. $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln(x+\sqrt{x^2+a^2}) + C$

[22]. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln|x+\sqrt{x^2-a^2}| + C$